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GENERAL SPATIAL STATIC CONTACT PROBLEM FOR A PRESTRESSED ELASTIC HALF-SPACE*

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The general spatial static contact problem for an elastic half-space with initial stresses is considered. Exact solutions are constructed for an arbitrary structure of the elastic potential, which are twice continuously differentiable functions of components of Green's strain tensor. The investigation is carried out in general form for compressible and incompressible bodies.

Questions associated with contact problems for bodies with initial stresses were examined in /1—4/ for particular forms of the elastic potential. Problems on the vibration of a rigid stamp on the surface of an initially stressed half-space and cylinder were examined from the aspect of the linearized theory of elastic wave propagation /7/ in /5, 6/. Contact problems for bodies with initial stresses were investigated within the framework of the linearized theory /7/ in /8, 9/ for an arbitrary structure of the elastic potential in general form for the theory of large (finite) initial strains and different modifications of the theory of small initial deformations. A formulation is given in /8, 9/ for contact problems for elastic bodies with initial stresses and torsion contact problems are examined. A number of contact problems for an elastic half-plane with initial stresses is examined in /10—12/ by using complex potentials of plane static linearized problems /13, 14/. Investigations are performed in general form for compressible and incompressible bodies. Complex potentials are introduced /15—18/ for plane dynamic problems and the plane dynamic contact problem for a prestressed half-plane is solved on their basis /19, 20/, when the initial problem allows transformation to a stationary problem in a moving coordinate system. The complex potentials introduced in the absence of initial stresses reduce to the complex S.G. Lekhnitskii potentials /21/ for an orthotropic linear elastic body in the case of unequal roots of the governing equation, and into the complex Kolosov-Muskhelishvili potentials for an isotropic linear elastic body in the case of equal roots.

1. Fundamental relationships.

We consider an infinite elastic isotropic or transversally-isotropic body with a homogeneous initial state of stress and strain governed by the expression

$$S_{11}^{\circ} = S_{22}^{\circ} \neq 0, \quad S_{33}^{\circ} = 0 \tag{1.1}$$

We introduce a Cartesian coordinate system y_1, y_2, y_3 and an arbitrary cylindrical coordinate system with axis Oy_3 in the initial state of strain by letting N and S denote the normal and tangent to the cylindrical surface. As in /23/, we introduce new variables $z_i = n_i^{-1/2} y_i$ for the cylindrical coordinate system. For the cases examined in /23/, it is proved that $\operatorname{Im} n_i = 0$ and $\operatorname{Re} n_i > 0$.

We consider two cases of the representation of solutions of spatial static problems for elastic bodies with initial stresses.

Equal roots. In the arbitrary cylindrical coordinate system we represent the displacements in the form (see (5.4) in /23/)

$$u_N = \frac{\partial}{\partial N}(\varphi_1 - \varphi_2) + z_1 \frac{\partial^2}{\partial N\, \partial z_1}\varphi_2 - \frac{\partial}{\partial S}\varphi_3 \tag{1.2}$$

$$u_S = \frac{\partial}{\partial S}(\varphi_1 - \varphi_2) - z_1 \frac{\partial^2}{\partial S\, \partial z_1}\varphi_2 + \frac{\partial}{\partial N}\varphi_3$$

$$u_3 = m_1 n_1^{-1/2}\left(\frac{\partial}{\partial z_1}\varphi_1 - z_1 \frac{\partial^2}{\partial z_1^2}\varphi_2\right) - (m_2 - 1) n_1^{-1/2} \frac{\partial}{\partial z_1}\varphi_2$$

For the stress vector terms whose components are referred to the body dimensions in the initial state of strain, we have for $y_3 = \text{const}$ /23/

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$$Q_{33}{}^* = c_{44}\left\{\frac{\partial^2}{\partial z_1^2}[(1+m_1)l_1\varphi_1 + (1+m_2)l_2\varphi_2] + (1+m_1)l_1z_1\frac{\partial^3}{\partial z_1^3}\varphi_2\right\} \quad (1.3)$$

$$Q_{3N}^* = c_{44}\left\{n_1^{-1/2}\frac{\partial^2}{\partial N\,\partial z_1}[(1+m_1)\varphi_1 + (1+m_2)\varphi_2] + n_1^{-1/2}(1+m_1)z_1\frac{\partial^3}{\partial N\,\partial z_1^2}\varphi_2 - n_3^{-1/2}\frac{\partial^2}{\partial S\,\partial z_3}\varphi_3\right\},$$

$$Q_{3S}^* = c_{44}\left\{n_1^{-1/2}\frac{\partial^2}{\partial S\,\partial z_1}[(1+m_1)\varphi_1 + (1+m_2)\varphi_2] + n_1^{-1/2}(1+m_1)z_1\frac{\partial^3}{\partial S\,\partial z_1^2}\varphi_2 + n_3^{-1/2}\frac{\partial^2}{\partial N\,\partial z_3}\varphi_3\right\}$$

Unequal roots. In the arbitrary cylindrical coordinate system we write the displacements in the form (see (4.7) in /23/)

$$u_N = \frac{\partial}{\partial N}(\varphi_1 - \varphi_2) - \frac{\partial}{\partial S}\varphi_3, \quad u_S = \frac{\partial}{\partial S}(\varphi_1 - \varphi_2) - \frac{\partial}{\partial N}\varphi_3 \quad (1.4)$$

$$u_3 = n_1^{-1/2}m_1\frac{\partial}{\partial z_1}\varphi_1 + n_2^{-1/2}m_2\frac{\partial}{\partial z_2}\varphi_2$$

We have for the stress vector components for $y_3 = \text{const}$ ($z_i = \text{const}$) /23/

$$Q_{33}{}^* = c_{44}\left[(1-m_1)l_1\frac{\partial^2}{\partial z_1^2}\varphi_1 - (1-m_2)l_2\frac{\partial^2}{\partial z_2^2}\varphi_2\right] \quad (1.5)$$

$$Q_{3N}^* = c_{44}\left[n_1^{-1/2}(1+m_1)\frac{\partial^2}{\partial N\,\partial z_1}\varphi_1 - n_2^{-1/2}(1-m_2)\frac{\partial^2}{\partial N\,\partial z_2}\varphi_2 - n_3^{-1/2}\frac{\partial^2}{\partial S\,\partial z_3}\varphi_3\right]$$

$$Q_{3S}^* = c_{44}\left[n_1^{-1/2}(1-m_1)\frac{\partial^2}{\partial S\,\partial z_1}\varphi_1 - n_2^{-1/2}(1-m_2)\frac{\partial^2}{\partial S\,\partial z_2}\varphi_2 - n_3^{-1/2}\frac{\partial^2}{\partial N\,\partial z_3}\varphi_3\right]$$

The functions φ_i $(i = 1, 2, 3)$ in (1.2)–(1.5) are defined by expressions presented in /23/. The quantities c_{44}, n_i, m_i, l_i $(i=1, 2)$ are defined in terms of the elastic potential and the initial state of stress and strain, respectively, for compressible and incompressible bodies by expressions presented in /23/.

2. Contact problem for an arbitrary contact domain. We will examine the fundamental results of an investigation for a prestressed elastic half-space $(y_3 < 0)$ subjected without friction to a stamp whose shape is determined by the function $u(y_1, y_2)$. We let S^* denote the contact domain for $y_3 = 0$, and $q(y_1, y_2)$ the normal pressure intensity acting outside the contact domain.

The boundary conditions of the problem for $y_3 = 0$ $(z_i = 0)$ have the form

$$u_3 = -u(y_1, y_2), \quad Q_{3N}{}^* = 0, \quad Q_{3S}{}^* = 0, \quad \forall (y_1, y_2) \in S^* \quad (2.1)$$

$$Q_{33}{}^* = -q(y_1, y_2), \quad Q_{3N}{}^* = 0, \quad Q_{3S}{}^* = 0, \quad \forall (y_1, y_2) \notin S^*$$

Here $u = u_0 + \alpha_1 y_1 + \alpha_2 y_2 + \alpha_0$, where $\alpha_i = \text{const}$ and are determined from the stamp equilibrium conditions

$$P_3 = \iint_{S^*} P\,dS^*, \quad M_j = \iint_{S^*} y_j P\,dS^*, \quad j = 1, 2, \quad P = -Q_{33}{}^*|_{y_3=0}$$

We perform the investigation in general form for compressible and incompressible bodies for an arbitrary structure of the elastic potential.

As in /23/, we introduce new potentials for equal roots $(n_1 = n_2)$ by the relationships

$$\varphi_1 = (1 - m_1)^{-1} f(y_1, y_2, z_1), \quad \varphi_2 = -(1 + m_2)^{-1} f(y_1, y_2, z_1) \quad (2.2)$$

$$\varphi_3 = 0$$

Taking account of (2.2), after some reduction we obtain from (1.2) and (1.3) (see (5.10) in /23/)

$$Q_{33}{}^* = c_{44}\left[\frac{\partial^2}{\partial z_1^2}(l_1 - l_2) - l_1\frac{1-m_1}{1+m_2}z_1\frac{\partial^3}{\partial z_1^3}\right]f \quad (2.3)$$

$$Q_{3L}^* = -\frac{c_{44}n_1^{-1/2}(1-m_1)z_1}{1+m_2}\frac{\partial^3}{\partial L\,\partial z_1^2}f, \quad L = N, S$$

$$u_3 = \frac{n_1^{-1/2}}{1-m_1}\left[\frac{1+2m_1-m_2}{1+m_2}\frac{\partial}{\partial z_1}f - m_1z_1\frac{\partial^2}{\partial z_1^2}f\right]$$

The function $f = f(y_1, y_2, z_1)$ is a solution of the equation

$$\frac{\partial^2 f}{\partial y_1^2} + \frac{\partial^2 f}{\partial y_2^2} + \frac{\partial^2 f}{\partial z_1^2} = 0 \tag{2.4}$$

For unequal roots $(n_1 \neq n_2)$ of the governing equation, we introduce potentials by the following relationships /23/:

$$\varphi_1 = \frac{\sqrt{n_1}}{1+m_1} f(y_1, y_2, z_1), \quad \varphi_2 = -\frac{\sqrt{n_2}}{1+m_2} f(y_1, y_2, z_2), \quad \varphi_3 = 0 \tag{2.5}$$

Taking account of (2.5), we have from (1.4) and (1.5) (see (4.17) from /23/)

$$\begin{aligned} u_3 &= \frac{m_1}{1+m_1} \frac{\partial}{\partial z_1} f(y_1, y_2, z_1) - \frac{m_2}{1+m_2} \frac{\partial}{\partial z_2} f(y_1, y_2, z_2) \\ Q_{33}^* &= c_{44} \left[\sqrt{n_1} l_1 \frac{\partial^2}{\partial z_1^2} f(y_1, y_2, z_1) - \sqrt{n_2} l_2 \frac{\partial^2}{\partial z_2^2} f(y_1, y_2, z_2) \right] \\ Q_{3L}^* &= c_{44} \frac{\partial}{\partial L} \left[\frac{\partial}{\partial z_1} f(y_1, y_2, z_1) - \frac{\partial}{\partial z_2} f(y_1, y_2, z_2) \right], \quad L = N, S \end{aligned} \tag{2.6}$$

where the function f is a solution of one of the equations

$$\frac{\partial^2 f}{\partial y_1^2} + \frac{\partial^2 f}{\partial y_2^2} + \frac{\partial^2 f}{\partial z_j^2} = 0, \quad j = 1, 2 \tag{2.7}$$

We examine the relationships (2.2)–(2.7) for $y_3 = 0$. In this case in place of (2.4) and (2.7), we can write one equation to determine the function f

$$\left(\frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial y_2^2} + \frac{\partial^2}{\partial y_3^2} \right) f(y_1, y_2, y_3) = 0 \tag{2.8}$$

Substituting (2.3) and (2.6) into (2.1), we obtain the following boundary conditions for the potential f for $y_3 = 0$:

$$\begin{aligned} A \frac{\partial f}{\partial y_3} &= -u(y_1, y_2), \quad \forall (y_1, y_2) \in S^* \\ B \frac{\partial^2 f}{\partial y_3^2} &= -q(y_1, y_2), \quad \forall (y_1, y_2) \notin S^* \end{aligned} \tag{2.9}$$

Here

$$A = \frac{1 + 2m_1 - m_2}{\sqrt{n_1}(1+m_1)(1+m_2)}, \quad B = c_{44}(l_1 - l_2) \quad (\text{case } (2.3)) \tag{2.10}$$

$$A = \frac{m_1 - m_2}{(1+m_1)(1+m_2)}, \quad B = c_{44}(\sqrt{n_1} l_1 - \sqrt{n_2} l_2) \quad (\text{case } (2.6)) \tag{2.11}$$

We introduce a new harmonic potential

$$A \partial f / \partial y_3 = V \tag{2.12}$$

Then for $y_3 = 0$ we obtain boundary conditions for the harmonic potential V from (2.9) and (2.12)

$$\begin{aligned} V &= -u(y_1, y_2), \quad \forall (y_1, y_2) \in S^*, \\ \frac{\partial V}{\partial y_3} &= -\frac{A}{B} q(y_1, y_2), \quad \forall (y_1, y_2) \notin S^* \end{aligned} \tag{2.13}$$

It follows from (2.13) and (1.2), say /24/, that the mixed problem for the potential (2.13) to which the contact problem reduces in the case of an elastic half-space with initial stresses, agrees in structure with the mixed problem to which the contact problem reduces for a half-space without initial stresses if we make the substitution (λ, μ are Lamé constants)

$$\frac{A}{B} \sim \frac{\lambda + 2\mu}{2\mu(\lambda + \mu)} \tag{2.14}$$

Therefore, the contact problem for an elastic half-space with initial stresses can be considered solved in the case when the contact problem of classical linear elasticity theory is solved for the corresponding contact domain. In this connection, known potentials (/24–27/, etc.) can be utilized for the contact problems of classical elasticity theory in the case of a half-space. Taking into account the substitution (2.14) and the representations of the solutions (2.3) and (2.6), the state of stress and strain of a half-space with initial stresses can be determined. If we are interested just in the pressure under the stamp $P = -Q_{33}^* |_{y_3=0}$ and the displacement of the half-space $u_3 |_{y_3=0}$, then several general assertions can be proved.

The expressions

$$u_3 = V(y_1, y_2, y_3), \quad \forall (y_1, y_2) \in S^*, \qquad P = -\frac{B}{A} \frac{\partial}{\partial y_3} V(y_1, y_2, y_3), \quad \forall (y_1, y_2) \in S^* \tag{2.15}$$

follow from (2.3), (2.6), (2.10)–(2.12) for $y_3 = 0$.

In the case when forces are not applied outside the stamp, i.e., $q(y_1, y_2) = 0$, it follows from (2.13)–(2.15) that the pressure distribution under the stamp for an elastic half-space with initial stresses differs from the corresponding distribution in classical elasticity theory by a factor dependent on the initial stress

$$K = \frac{B}{A} \frac{\lambda + 2\mu}{2\mu(\lambda + \mu)} \tag{2.16}$$

Analogous assertions are obtained in /10, 11/ for an elastic half-plane with initial stresses.

The result formulated above affords the possibility of determining the distribution of the state of stress just under the stamp by means of the known solution for an elastic half-space without initial stresses in the case of an arbitrary contact domain. The problem of determining the state of stress and strain in the whole half-space reduces to seeking the harmonic functions $f(y_1, y_2, y_3)$ when it is further substituted into the expressions for the displacements and stresses. The results obtained above were published partially in /28/.

3. **The axisymmetric contact problem.** As an example of the reduction of the mixed problem with initial stresses to the classical mixed problem of potential theory, we consider the axisymmetric contact problem for a prestressed half-space. We limit ourselves here to the case of multiple roots ($n_1 = n_2$) of the governing equation. This case holds, for instance, for compressible bodies with an harmonic-type potential and for incompressible bodies with a Bartenev-Khazanovich potential /23/. In the axisymmetric case under consideration we have from (1.2), (2.2) and (2.3) for determining the displacement in terms of the harmonic function $f = f(r, z_1)$ in a circular cylindrical r, θ, y_3 coordinate system

$$u_r = \frac{m_2 - m_1}{(1 + m_1)(1 + m_2)} \frac{\partial f}{\partial r} - (1 + m_2)^{-1} z_1 \frac{\partial^2 f}{\partial r\, \partial z_1} \tag{3.1}$$

$$u_3 = \frac{n_1^{-1/2}}{1 + m_1} \left[\frac{1 + 2m_1 - m_2}{1 + m_2} \frac{\partial f}{\partial z_1} - m_1 z_1 \frac{\partial^2 f}{\partial z_1^2} \right]$$

From (2.3) we derive expressions for the stress vector components for $y_3 = \mathrm{const}$ ($z_i = \mathrm{const}$) in the form

$$Q_{3r}^* = -\frac{c_{44}(1 + m_1) z_1}{(1 + m_1)(1 - m_2)} \frac{\partial^3 f}{\partial r\, \partial z_1^2} \tag{3.2}$$

$$Q_{33}^* = c_{44} \left[(l_1 - l_2) \frac{\partial^2}{\partial z_1^2} - \frac{l_1(1 + m_1) z_1}{1 + m_2} \frac{\partial^3}{\partial z_1^3} \right] f$$

The function $f = f(r, z_1)$ is determined from the equation

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial z_1^2} = 0$$

Let a stamp bounded by a surface of revolution be impressed in an elastic prestressed half-space $z \leqslant 0$. It is assumed that there are no friction forces between the stamp and the half-space and that a given external load P_3 directed along the axis of symmetry acts on the stamp.

The boundary conditions of the problem have the following form for $y_3 = 0$

$$u_3(r, 0) = -\varphi(r), \quad Q_{3r}^*(r, 0) = 0, \quad 0 \leqslant r < a \tag{3.3}$$

$$Q_{33}^*(r, 0) = 0, \quad Q_{3r}^*(r, 0) = 0, \quad r > a$$

where $\varphi(r) = \varphi_0(r) - \alpha_0$ ($\varphi_0(r)$ is the function determining the shape of the stamp, α_0 is the translational displacement of the stamp, and a is the radius of the contact area).

For $y_3 = 0$ ($y_3 \equiv z_1 = 0$), taking account of (3.1) and (3.2), we obtain the boundary conditions

$$\frac{\partial f}{\partial y_3} = -\frac{\varphi(r)}{A}, \quad A = \frac{1 + 2m_1 - m_2}{(1 + m_1)(1 + m_2)\sqrt{n_1}}, \quad 0 \leqslant r < a \tag{3.4}$$

$$\partial^2 f / \partial y_3^2 = 0, \quad r > a$$

to determine the function $f = f(r, y_3)$ that is a solution of the equation

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial y_3^2} = 0 \tag{3.5}$$

The mixed problem for a harmonic potential (3.4), (3.5) to which the contact problem for an elastic half-space with initial stresses reduces is in complete agreement with the corresponding problem in the case of no initial stresses /29/ if the following substitution is introduced

$$A = -\frac{\lambda + 2\mu}{\lambda + \mu}$$

Following /29, 30/ we obtain from (3.4) that the state of stress and strain in a half-space is characterized by the following functions:

$$f(\rho, y_3) = \int_0^\infty p^{-1} C(p) J_0(p\rho) \exp\left(-\frac{p}{a} y_3\right) dp \tag{3.6}$$

$$C(p) = -\frac{2}{\pi A}\left\{\cos(p)\int_0^1 y(1-y^2)^{-1/2}\varphi(y)\,dy + \int_0^1 y(1-y^2)^{-1/2}\,dy\int_0^1 \varphi(yx)\,px\sin(px)\,dx\right\}$$

$$(C(p) = C_1(p/a),\ r = a\rho,\ p = as,\ 0 \leqslant \rho \leqslant 1)$$

Taking into account that here $z_i = y_3/\sqrt{n_i}$, $i = 1, 2$, on the basis of (3.1), (3.2) and (3.6) we can determine the stress and displacement distributions in the half-space.

Although the mixed problem for the potential to which the contact problem reduces in the case of an elastic half-space with initial stresses (under the substitution mentioned) agrees with the corresponding mixed problem for a half-space without initial stresses, all the states of stress and strain will be different in the half-space for the problems mentioned. The latter is associated with the fact that for these problems the displacments and stresses are represented in terms of harminic functions of their arguments in a different manner.

4. Example. We determine the influence of the initial stresses on the pressure distribution under the stamp that acts on a prestressed elastic half-space. Denoting the pressure under the stamp by P, P^c, respectively, for the prestressed half-space and in the case of no initial stresses, we have $P = P^c K$ according to the assertion in Sect.2, where K is defined by (2.16).

In the case of an incompressible body with Bartenev-Khazanovich potential, we have for the constants /23/

$$c_{44} = 2\mu\lambda_1^{-2}(\lambda_1^3 + 1)^{-1},\ \lambda_3 = \lambda_1^{-2},\ n_1 = n_2 = \lambda_1^{-3},\ m_1 = \lambda_1^{-3}$$
$$l_1 = \lambda_1^3,\ m_2 = 1,\ l_2 = {}^1/_2(1 - \lambda_1^3) \tag{4.1}$$

For an incompressible body $\lambda \to \infty$, consequently, taking account of (2.16) and (4.1), we obtain

$$K = \frac{3\lambda_1^3 - 1}{2\lambda_1^3\sqrt{\lambda_1}} \tag{4.2}$$

For an incompressible body with Treloar potential, the constants have the form

$$c_{44} = 2c_{10}\lambda_1^{-4},\ \lambda_3 = \lambda_1^{-2},\ n_1 = \lambda_1^{-6},\ n_2 = 1,\ m_1 = \lambda_1^{-6}$$
$$m_2 = 1,\ l_1 = 2\lambda_1^6(1 + \lambda_1^6)^{-1},\ l_2 = {}^1/_2(1 + \lambda_1^6) \tag{4.3}$$

For an incompressible body with Treloar potential we should set $\mu = 2c_{10}$, $\lambda \to \infty$ hence, we obtain from (2.16) and (4.3)

$$K = \frac{\lambda_1^9 + \lambda_1^6 - 3\lambda_1^3 - 1}{2\lambda_1^4(1 - \lambda_1^3)} \tag{4.4}$$

We present below, for certain values of λ_1, the corresponding values of K for a body with Treloar potential and (in the parentheses) for a body with Bartenev-Khazanovich potential

λ_1	0,67 (0,69)	0,8	1	1,5	2
K	0 (0)	0.75 (0.55)	1 (1)	1,33 (1,12)	2,08 (1,02)

It is seen that the influence of the initial stresses on the pressure distribution under the stamp is sufficiently substantial in the case of incompressible bodies.

For $\lambda_1^* \approx 0.67$ (0.69) (this corresponds to a surface instability of the half-space), we obtain $K = 0$ from (4.2) and (4.4), i.e., the pressure under the stamp is zero. An analogous phenomenon was detected for the torsion contact problems /8/ and the plane problems /10, 11/. From the physical viewpoint it is natural that an achieving the initial state of values that correspond to the surface instability, it is almost not necessary to apply forces for small stamp displacements (within the framework of the linearized theory).

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ON THE UNLOADING PROCESS FOR CONTACT INTERACTION*

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The unloading process in a body under the action of a stamp is investigated. It is assumed that the unloading occurs at all points of the body. The contact area between the body and the stamp can change during the unloading; consequently, the unloading problem during contact interaction is non-linear. A generalization to the case of contact problems is proposed for the theorem of unloading /1/. A variational principle is obtained in the unloading displacements, and the existence and uniqueness of the solution of the unloading problem are investigated. The unloading process is examined in an elastic-plastic half-space on which a stamp of circular planform acts. The change in the contact area and in the contact stresses during unloading is studied, and the shape of the residual impression is obtained. The problem is investigated by using the Galin solution /2/ of the action of a circular stamp and a load applied outside the stamp on an elastic half-space. Numerical methods of solving contact problems with unloading are also examined; an example is presented for the numerical solution of the problem of plane deformation in the compression of a strip by two stamps with subsequent unloading.

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